The data of other investigators [4, 5, 6] are also generalized with the help of Eq. (11) (Fig. 4). Thus, the data from [4, 6] are generalized by this dependence within the limits of of ± 25% (Fig. 4), while the experimental points obtained from the data of [5] are located below the calculated curve by an average of 20-30%.

NOTATION

d, tube diameter, mm; c_p, specific isobaric heat capacity, J/mole·°K; P, pressure, N/m²; q_w, heat-flux density at wall, W/m²; T, temperature, °K; ΔT, local temperature head, °K; T_m, pseudocritical temperature, °K; i, enthalpy, J/mole; α_f, local heat-transfer coefficient, W/m²·°K; $\overline{\rho w}$, mass velocity, kg/m²·sec; T_s/T_m, reduced temperature of stream core; β, coefficient of volumetric expansion, $°K^{-1}$; F, correction factor; H', h^+, dimensionless numbers. Indices: w, parameters of wall; s, parameters of stream core; 0, initial conditions; cr, critical

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HEAT TRANSFER BETWEEN ROTATING DISKS IN A CLOSED CAVITY

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UDC 536.24

The heat transfer of a disk in a closed rotating cavity with laminar and turbulent boundary layers is determined experimentally.

Information on heat transfer in closed rotating cavities is required when estimating the temperature state of the rotors of gas and steam turbines, electrical machinery, etc. The theoretical solution of this problem for the self-similar case of laminar convection has been examined in [1] and for the case when the laminar or turbulent boundary layers at the surfaces of the disks are separated by the core of the stream — in [2, 3].

The theoretical solutions are obtained with the use of a number of simplifying premises. Moreover, mass forces alter the boundaries of the modes of flow [4] and a theoretical analy-

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sis is not yet capable of predicting these changes. All this makes it necessary to study this problem experimentally.

A schematic diagram of the experimental installation is shown in Fig. 1. The experimental cavity is formed by two disks 1 which are 930 mm in diameter and 25 mm thick, a spacing ring 2, and a pipe 3. Inner dimensions of rotating cavity: maximum and minimum diameters 834 mm and 81 mm, width l = 50 mm. The annular gap 12 with a radial dimension of 2 mm is designed for filling the cavity with water. About 25% of this gap by area is blocked by projections 13 of the pipe, which practically eliminates the return flow of liquid through this gap during an experiment. The material of the disks is ÉI 481 and that of the ring is ÉI 417. The rotor assembly is set into rotation through the reducer 4 by a direct current motor with smooth regulation of the rpm. Through the shaft of the multiplier–reducer 5 the rotation is conveyed to the rpm pickups 6 of the tachometers 7 and 8 (ITÉ-1 and TSFU1-2) while it is conveyed from the rotor to the current pickups 9. The contact of the graphite brushes of the current pickups with the rings is provided by a pneumatic system.

The heating of one of the disks was accomplished using the electric furnace 10 through the air gap 11. The spiral furnace assures uniform heat release over the area.

The local heat-exchange coefficients at the surface of the "hot" disk were determined on the basis of the gradient method. For this purpose 34 copper–Constantan thermocouples were set on the surface of each disk along a closed contour located in a plane passing through the axis of rotation. The thermocouple readings were recorded with three modified 24-point ÉPP-09 instruments. The temperature gradients at the heat-exchange surfaces were determined on the basis of the calculation of the temperature field of the disk by the numerical method on an M-222 computer with allowance for the temperature dependence of the heat capacity and thermal conductivity of the disk material.

The heat-exchange coefficients were measured in a quasisteady mode of free convection. In accordance with [5] the quasisteadiness of the mode was verified from the condition

$$\Theta = \frac{c''_{\omega}}{\rho c_j L M} (b \, \mathrm{Gr}^*_{\infty} \mathrm{Pr})^{1/5} > 1. \tag{1}$$

The relative width l/R = 0.12 of the experimental cavity (R is the maximum radius of the cavity in meters) assured the absence of merging of the boundary layers. In experiments with a closed, rectangular, stationary cavity [6] merging of the boundary layers was not observed even with a relative width of 0.025, while for flow around a disk in a housing without a duct [7] it was not observed with $l/R \geqslant 0.1$.

The results of the experimental study are generalized in the form of the dependence $\overline{\mathrm{Nu}} = f(\overline{\mathrm{Ra}}^*_R)$, where $\overline{\mathrm{Ra}}^*_R$ is the average modified Rayleigh number, which in accordance with [6] is determined by the expression

$$\overline{\mathrm{Ra}}^*_R = \overline{\mathrm{Gr}}^*_R \mathrm{Pr} = \overline{\mathrm{Gr}}_R \overline{\mathrm{Nu}} \mathrm{Pr} = \frac{jR^3}{\nu^2} \beta \overline{\Delta t} \, \overline{\mathrm{Nu}} \, \mathrm{Pr}, \tag{2}$$

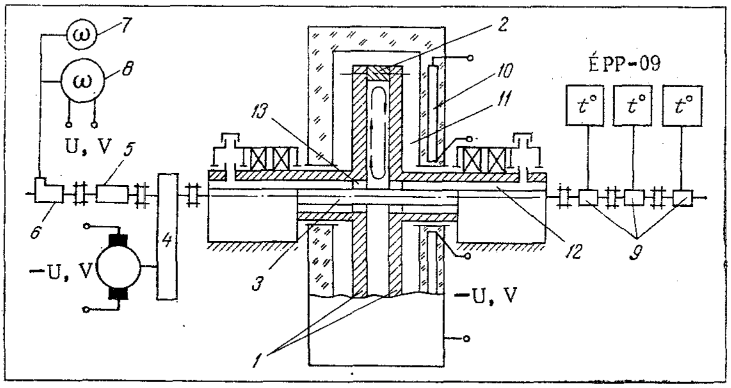

Fig. 1. Diagram of experimental installation.

TABLE 1. Experimental Data on Heat Transfer

ω, $\frac{\text{rad}}{\text{sec}}$	$\bar{t}_{w1}$, °C	$\bar{t}_{w2}$, °C	$\bar{\alpha}$, $\frac{\text{W}}{\text{m}^2\cdot\text{deg}}$	Nu	$\overline{Ra}^*_R$
20,93	23,92	19,45	47,00	32,47	$3{,}46\cdot10^{12}$
31,4	27,06	22,05	56,47	38,70	$1{,}26\cdot10^{13}$
20,93	41,50	32,87	53,14	35,23	$1{,}65\cdot10^{13}$
31,4	51,71	24,06	78,39	50,95	$2{,}33\cdot10^{14}$
20,93	41,42	21,12	60,31	39,99	$4{,}4\cdot10^{13}$
20,93	56,08	23,38	67,68	43,66	$1{,}17\cdot10^{14}$
20,93	45,71	33,78	53,05	34,87	$2{,}59\cdot10^{13}$
41,9	33,88	23,48	68,77	46,36	$7{,}8\cdot10^{13}$
41,9	44,35	24,71	80,71	53,20	$2{,}49\cdot10^{14}$
67,99	36,81	20,38	61,68	41,30	$3{,}27\cdot10^{14}$
67,99	48,06	24,60	102,75	67,23	$1{,}097\cdot10^{15}$
67,99	57,83	33,67	168,10	108,12	$2{,}36\cdot10^{15}$
115,13	33,12	19,58	117,00	79,00	$1{,}27\cdot10^{15}$
115,13	43,93	25,13	141,06	93,05	$3{,}12\cdot10^{15}$
115,13	70,15	53,39	280,92	177,37	10^{16}
146,53	31,66	24,36	104,45	70,77	$9{,}27\cdot10^{14}$
146,53	39,74	28,12	123,95	82,47	$2{,}4\cdot10^{15}$
146,53	59,03	44,22	213,09	136,79	$8{,}73\cdot10^{15}$
209,33	36,71	30,45	151,53	101,49	$2{,}88\cdot10^{15}$
209,33	47,88	36,57	238,70	156,25	$1{,}17\cdot10^{16}$
209,33	59,21	46,64	369,29	237,00	$2{,}62\cdot10^{16}$
230,27	47,19	38,98	313,82	205,70	$1{,}33\cdot10^{16}$
230,27	51,12	42,81	390,96	254,40	$1{,}85\cdot10^{16}$

where $\overline{\Delta t} = \bar{t}_{w1} - \bar{t}_{w2}$ is the difference, averaged over the heat-exchange area, between the temperatures of the "hot" and "cold" disks, characterizing the driving force of convection in the cavity.

The thermophysical properties of the liquid are determined for its average temperature while the centrifugal acceleration j, m/sec^2, is determined at the maximum radius of the cavity.

The heat-exchange coefficient entering into the Nusselt number is determined by the equation

$$\bar{\alpha} = \bar{q}/\overline{\Delta t}.$$

Modes of heat exchange in which ω = 20-230 rad/sec and q = 0-5·10^3 W/m^2 ($\overline{Ra}^*_R$ = 10^{12}-$3.16\cdot10^{16}$ and $\overline{Gr}_R/Re^2$ = 0-0.02) were studied.

The experiments were performed in the following order: after the installation was prepared with the rotor stationary the potentiometers for the continuous recording of the temperatures at 23 points of the surfaces of each disk were turned on, then the rotor was taken up to the chosen rotation speed, and after a certain time interval the oven was turned on.

The results of the study are presented in Table 1 and Fig. 2. As seen from the figure, at a constant rotation speed the heating of the rotating liquid first leads to a decrease in the heat-exchange coefficient and then to its increase. This can be explained by the fact that at the start of the heating stage the temperature heads are small and do not produce thermal convection, rather the heat is transferred to the liquid by thermal conduction. The further increase in the temperature head leads to the appearance of convective motion with the formation at the surfaces of the disks of boundary layers, laminar or turbulent, depending on the rotation speed and the heat-flux density. At this stage of heating of the liquid the dependence $\overline{Nu} = f(\overline{Ra}^*_R)$ becomes the same for different rotation speeds.

The results of the experiments with a developed boundary layer at the surfaces of the disks are generalized by the similarity equations

$$\text{for } \overline{Ra}^*_R = 10^{12} - 2\cdot10^{14},\ \overline{Nu} = 1.82\,\overline{Ra}^{*\,0.1}_R, \tag{3}$$

$$\text{for } \overline{Ra}^*_R = 2\cdot10^{14} - 3.16\cdot10^{16},\ \overline{Nu} = 0.000398\,\overline{Ra}^{*\,0.35}_R. \tag{4}$$

After the substitution of $\overline{Ra}^*_R$ from (2), Eqs. (3) and (4) are reduced to the form

$$\overline{Nu} = 1.07\,\overline{Ra}_R^{0.111}, \tag{5}$$

$$\overline{Nu} = 0.00000589\,\overline{Ra}_R^{0.539}. \tag{6}$$

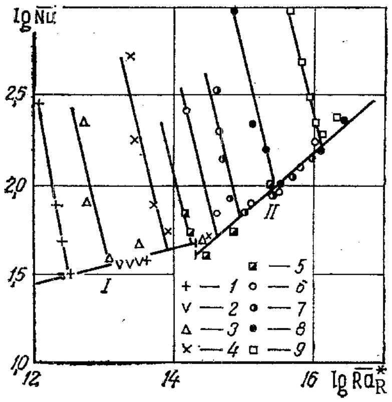

Fig. 2

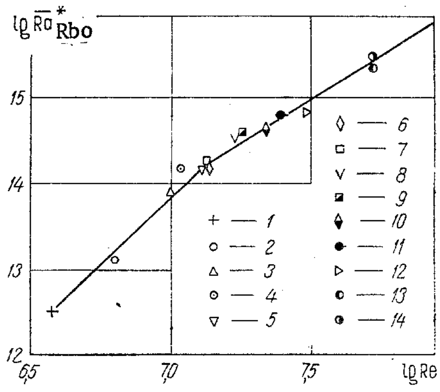

Fig. 3

Fig. 2. Results of generalization of experiments with a developed boundary layer: 1) ω = 20.93 rad/sec; 2) 20.93; 3) 31.4; 4) 52.33; 5) 67.99; 6) 115.13; 7) 146.53; 8) 209.33; 9) 230.27.

Fig. 3. Dependence of $\overline{Ra}^*_{Rbo}$ on Re: 1) ω = 20.93 rad/sec; 2) 31.4; 3) 41.9; 4) 41.9; 5) 52.33; 6) 52.33; 7) 52.33; 8) 67.99; 9) 88.97; 10) 115.13; 11) 115.13; 12) 146.53; 13) 209.33; 14) 209.33.

A comparison of the powers on the numbers $\overline{Ra}_R$ in Eqs. (5) and (6) with the known experimental functions for heat transfer with free convection in gravitational and inertial mass force fields allows one to assume that the dependence (5) characterizes the heat transfer with a laminar, and the dependence (6) with a turbulent, boundary layer. Consequently, for the conditions under consideration the formation of a turbulent boundary layer occurs with $\overline{Ra}_{Rcr} = 4.17 \cdot 10^{12}$.

The boundary between the conductive and convective mechanisms of heat exchange between the surfaces of the disks and the liquid, determined by the quantity $\overline{Ra}^*_{Rbo}$, depends on the rotation speed. The dependence $\overline{Ra}^*_{Rbo} = f(Re)$, ($Re = \omega R^2/\nu$), constructed with the help of Fig. 2, is shown in Fig. 3. As seen from the figure, the quantity $\overline{Ra}^*_{Rbo}$ increases with an increase in the rotation speed, which indicates the stabilizing effect of rotation on the movement of the liquid.

In Fig. 4a the results of a study of the heat transfer of water in a closed rotating gap (lines I and II, similarity equations $\overline{Nu} = 1.07\overline{Ra}_R^{0.111}$, $\overline{Nu} = 0.00000589\overline{Ra}_R^{0.539}$) are compared with the results of an experimental study of heat transfer in a stationary, closed, rectangular cavity with laminar and turbulent boundary layers (lines III and IV, similarity equations $\overline{Nu} = 0.42\ \overline{Ra}_H^{0.25}\ Pr^{0.012}(H/l)^{-0.05}$, $\overline{Nu} = 0.046\ \overline{Ra}_H^{1/3}$) [6] and with the results of an experimental study at a vertical wall in a large volume with laminar and turbulent boundary layers (lines V and VI). We assume that the ratio of the average heat-transfer intensity to the local value equals 1.25 in the laminar mode and 1 in the turbulent mode, when the similarity equations have the form $\overline{Nu} = 0.278\ \overline{Ra}_H^{0.25}$, $\overline{Nu} = 0.1996\ \overline{Ra}_H^{0.282}$ [8]. The experiments, described in [6] and [8], were performed on water with q = const.

In Fig. 4a it is seen that with $\overline{Ra}_R$ = idem the heat transfer occurs less intensely at the surface of a rotating disk than at a stationary surface. This is due primarily to the later transition from the laminar to the turbulent boundary layer at a rotating surface compared with a stationary surface and to the stabilizing effect of the inertial mass forces on the flow. Some differences between the functions $\overline{Nu} = f(\overline{Ra}_R)$ at the rotating disk and at the stationary surface are connected with the method of averaging the heat-exchange coefficient on the surface of the disk and with the choice of the maximum value of the inertial acceleration as characteristic.

Other reports have also indicated the stabilizing role of centrifugal mass forces. For example, in a study of friction and the conditions of the transition from laminar to turbulent flow this effect was observed in radial rotating channels [9] and in pipes when rotated about their own axis [4].

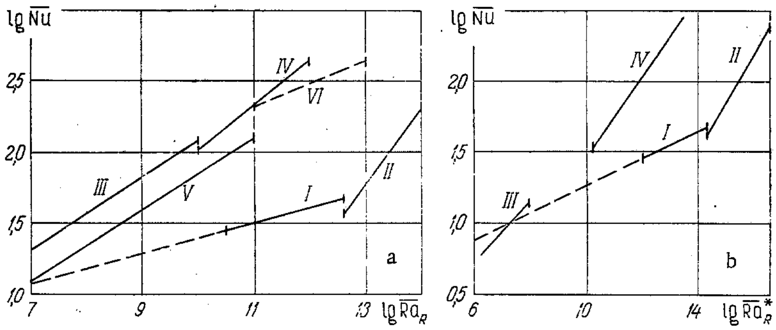

Fig. 4. Comparison of results obtained with literature data.

The comparison of the experimental results obtained with the theoretical results of Kapinos [2, 3] was made under the assumption that the effect of the physical properties of the liquid on the heat-exchange process is taken into account by the Prandtl number, which enters into the Rayleigh number. The validity of this assumption for laminar flow in the region of $Pr \geqslant 0.7$ is confirmed theoretically and experimentally; in turbulent flow an additional dependence of the constant c in the similarity equation $\overline{Nu} = c(\overline{Gr}_H Pr)^m$ on the Prandtl number is predicted in some theoretical reports, but sufficient proof of this assumption has not yet been obtained. In the units compared, with allowance for the assumptions made, Kapinos's equations for laminar and turbulent boundary layers have the form (the heat-transfer intensity in the laminar mode at the "hot" and "cold" disks is assumed to be the same)

$$\overline{Nu} = 0.353\, \overline{Ra}_R^{*0.2}, \tag{7}$$

$$\overline{Nu} = 0.0364\, \overline{Ra}_R^{*0.286}. \tag{8}$$

In Fig. 4b, lines I and II correspond to Eqs. (3) and (4) and lines III and IV to Kapinos's solutions for laminar and turbulent boundary layers.

Extrapolation of the experimental data obtained with a laminar boundary layer to the region of low Rayleigh numbers shows that the true values of the number $\overline{Nu}$ are close to those of the theoretical equation of Kapinos. The true Nusselt numbers with a turbulent boundary layer are considerably less than the theoretical values calculated from Kapinos's equation because of the stabilizing effect of the centrifugal mass forces on the free motion of the liquid.

NOTATION

l, width of cavity, m; $\bar{\alpha}$, average heat-exchange coefficient, $W/m^2 \cdot deg$; q, heat-flux density, W/m^2; $\bar{q}$, average heat-flux density, W/m^2; $\overline{Gr}_R^*$, average modified Grashof number; acceleration for stationary systems equals free-fall acceleration; Pr, Prandtl number; $\overline{Gr}_R$, average Grashof number; Re, Reynolds number; $\overline{Ra}_{Rcr}$, critical Rayleigh number of transition in modes of flow; ω, angular velocity, rad/sec; ν, coefficient of kinematic viscosity, m^2/sec; c, constant in a similarity equation; m, exponent in a similarity equation; β, coefficient of volumetric expansion, 1/deg; $\overline{Ra}_R$, average Rayleigh number, equal to $\overline{Gr}_R Pr$; H, height of wall or rectangular cavity, m; $\overline{Ra}^*_{Rbo}$, boundary Rayleigh number determining the transition from conductive to convective heat exchange; $\Delta\bar{t}$, difference in wall temperatures for rotating and stationary cavities or twice the temperature difference between wall and liquid for a vertical wall in a large volume, °K; Θ, generalized variable of heat capacity of wall; c_f, heat capacity of liquid; ρ, density of liquid; c_w'', heat capacity of wall per unit surface area; L, height of wall; M, derivative of generalized temperature distribution, tabulated in [5]; b, coefficient of time normalization, tabulated in [5]; Gr_∞^*, modified Grashof number calculated from L and heat flux corresponding to stationary mode; $\overline{Nu}$, average Nusselt number, equal to $\alpha\bar{R}/\lambda$ or $\alpha\bar{H}/\lambda$; λ, coefficient of thermal conductivity, W/m·deg.

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EFFECT OF TEMPERATURE AND SLOPE OF A GAS MAIN IN NONISOTHERMAL UNSTEADY GAS MOVEMENT

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UDC 533.6

The effect of the temperature and slope of a pipeline on the variation in gasdynamic parameters is studied using numerical methods.

The solution of the problem of the unsteady movement of a real gas through a long gas main has great theoretical and practical importance. The problem becomes relatively complicated when the nonisothermy and slope of the gas main are taken into account. The effect of the slope of the gas main on the parameters of the moving gas becomes important in mountainous terrain or when the gas is fed through a long gas main of large diameter with a slope which varies (even slightly) from horizontal.

Many reports have been devoted to these problems. Numerous linearized solutions of the problem of unsteady movement of a real gas through long gas mains are known ([1-9] and others).

The case of a vertical pipe set in natural soil, which occurs when gas is extracted from great depths, is of particular interest.

The unsteady nonisothermal movment of a gas through a long pipeline is examined below.

1. Differential Equations of Motion. Initial and Boundary Conditions

Let us consider the one-dimensional, unsteady, nonisothermal motion of a gas in a long gas main. Such gas motion can be described by the system of differential equations [1]

$$-\frac{\partial p}{\partial x}=\frac{\lambda\rho u^2}{8\delta}+\rho g\sin\alpha;$$

$$-\frac{\partial \rho}{\partial t}=\frac{\partial}{\partial x}\ (\rho u); \tag{1.1}$$

$$p=\rho gRT;\ G=gs\rho u.$$

It is assumed that the gas temperature varies along the gas main as a function of the coordinate and is a function assigned in advance.

Placing the origin of coordinates at the start of the pipe, directing the axis along the length of the pipe, and assuming for determinacy that the gas temperature along the pipeline varies by a linear law, one can express T(x) by the following equation [10]:

$$T(x)=T_s-(T_s-T_e)\ \frac{x}{L}, \tag{1.2}$$

Erevan Zootechnical-Veterinary Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 30, No. 4, pp. 619-624, April, 1976. Original article submitted April 15, 1975.